Accuracy of Admissible Heuristic Functions in Selected Planning Domains

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Motivation

Goal

- Goal: Find accurate admissible heuristics
- Subgoal: Assess accuracy of known heuristics

Approaches

- Popular: Run planners on benchmarks, count node expansions
  Drawback: Only comparative statements
- Alternative: Analytically compare heuristics to optimal heuristic on benchmark domains
  Advantage: Absolute statements

Performance Measure

Comparison to h^*

- Comparison of heuristics to h^*
- Derivation of domain-dependent worst-case bounds on accuracy
- Establishment of tight bounds

Formal Measure

For fixed heuristic h and domain $\mathcal{D}$, a value $\alpha = \alpha(h, \mathcal{D}) \in [0, 1]$ s.t.

$$h(s) \geq \alpha h^*(s) + o(h^*(s)) \quad \text{for all states } s, \text{ and}$$

$$h(s_n) \leq \alpha h^*(s_n) + o(h^*(s_n)) \quad \text{for a family of solvable states } (s_n)_{n \in \mathbb{N}} \text{ s.t. } h^*(s_n) \overset{n \to \infty}{\longrightarrow} \infty$$

Heuristics and Planning Domains

Compared Heuristics

- h^* optimal plan length
- h^+ optimal relaxed plan length
- h^k cost of most costly size-k goal subset
- h^{PDB} pattern database heuristics
- $h^{\mathrm{PDB}}_{\mathrm{add}}$ additive pattern database heuristics

Considered Planning Domains

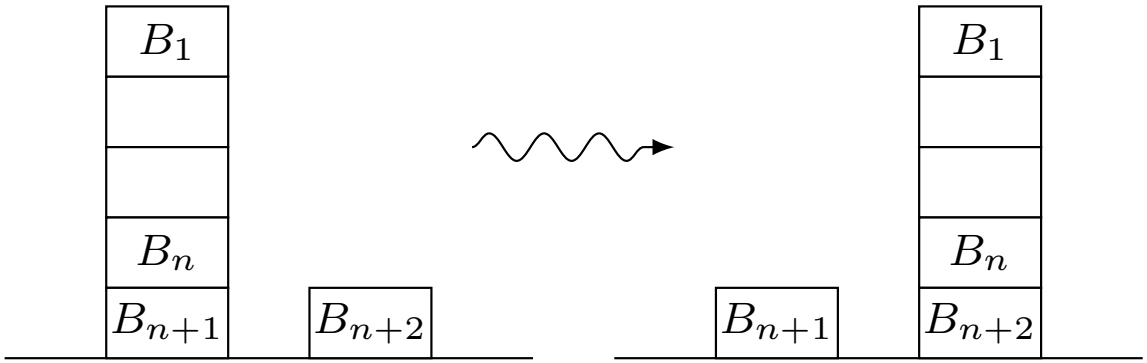
GRIPPER, LOGISTICS, BLOCKSWORLD, MICONIC-STRIPS, MICONIC-SIMPLE-ADL, SCHEDULE, SATELLITE

The h^+ Heuristic

Example (BLOCKSWORLD)

$m =$ number of blocks touched in optimal plan

$h^*(s) \leq 4m,\ h^+(s) \geq m \Longrightarrow \alpha(h^+, \text{BLOCKSWORLD}) \geq 1/4$



$h^*(s_n) = 4n - 2,\ h^+(s_n) = n + 1 \Longrightarrow \alpha(h^+, \text{BLOCKSWORLD}) \leq 1/4$

$$\boxed{\alpha(h^+, \text{BLOCKSWORLD}) = 1/4}$$

The h^k Heuristic Family

Arbitrary Inaccuracy

- For all domains $\mathcal{D}$ investigated:

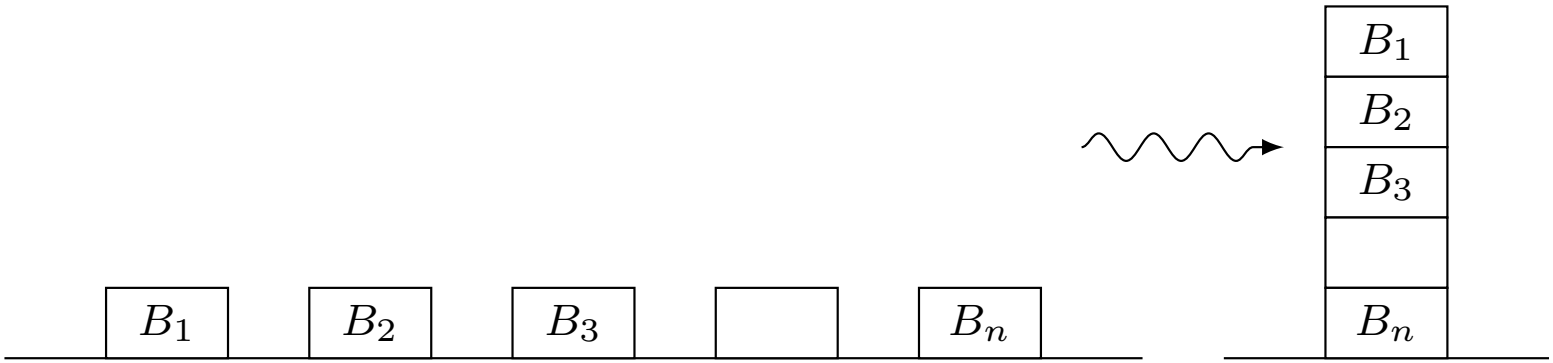
$$\alpha(h^k, \mathcal{D}) = 0.$$

- Reason: there are families of states $(s_n)_{n \in \mathbb{N}}$ s.t.

$$h^*(s_n) \in \Omega(n) \quad \text{but}$$
$$h^k(s_n) \in O(1) \quad (h^k(s_n) \in O(k))$$

The h^k Heuristic Family

Example (BLOCKSWORLD)



$$h^*(s_n) = 2n - 2,\ h^k(s_n) = 2k$$

$$\alpha(h^k, \text{BLOCKSWORLD}) = 0$$

(Additive) Pattern Database Heuristics

Pattern Size Restriction

- Let n be the problem size.
- Bounded memory: Database size limit $O(n^k)$.
- Consequently: Pattern size limit $O(\log n)$ variables.

Non-Additive Pattern Database Heuristics

Arbitrary Inaccuracy

- For all domains $\mathcal{D}$ investigated:

$$\alpha(h^{\mathrm{PDB}}, \mathcal{D}) = 0.$$

- Reason: there are families of states $(s_n)_{n \in \mathbb{N}}$ s.t.

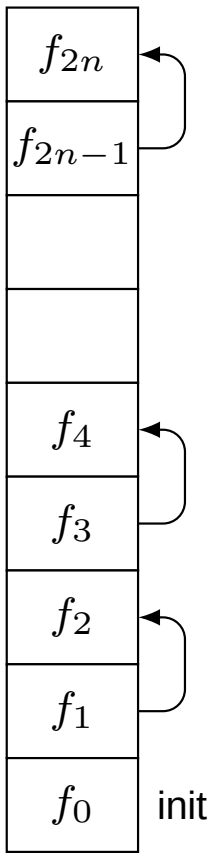
$$h^*(s_n) \in \Omega(n) \quad \text{but}$$
$$h^{\mathrm{PDB}}(s_n) \in O(\log n) \quad \text{since}$$

  - only $k(n) \in O(\log n)$ variables in pattern,
  - at most $k(n)$ goals in abstract problem,
  - $h^{\mathrm{PDB}}(s_n) \leq h^{k(n)}(s_n) \in O(k(n)) = O(\log n)$.

Additive Pattern Database Heuristics

Example: MICONIC-STRIPS

Example (MICONIC-STRIPS)



- Singleton patterns for goal variables
  $\rightsquigarrow \alpha(h^{\mathrm{PDB}}_{\mathrm{add}}, \text{MICONIC-STRIPS}) \geq 1/2$.
- $h^*(s_n) = 4n$.
- State vars: $\texttt{elev}, \texttt{pass}_1, \ldots, \texttt{pass}_n$
- Optimal additive PDB:
  - pattern $\{\texttt{elev}, \texttt{pass}_1, \ldots, \texttt{pass}_K\}$, $K \in O(\log n)$,
  - patterns $\{\texttt{pass}_{K+1}\}, \ldots, \{\texttt{pass}_n\}$
- $h^{\mathrm{PDB}}_{\mathrm{add}}(s_n) = 4K + 2(n - K) = 2n + 2K$.
- $\alpha(s_n) = (2n + O(\log n))/4n \to 1/2 \quad (n \to \infty)$.

$$\alpha(h^{\mathrm{PDB}}_{\mathrm{add}}, \text{MICONIC-STRIPS}) = 1/2$$

Summary of Performance Ratios

Performance Ratios

Domain	Heuristic h^+	h^k	h^{PDB}	$h^{\mathrm{PDB}}_{\mathrm{add}}$
GRIPPER	$2/3$	0	0	$2/3$
LOGISTICS	$1/2$–$3/4$	0	0	$1/2$
BLOCKSWORLD	$1/4$	0	0	0
MICONIC-STRIPS	$6/7$	0	0	$1/2$
MICONIC-SIMPLE-ADL	$3/4$	0	0	open
SCHEDULE	$1/4$	0	0	$1/2$
SATELLITE	$1/2$	0	0	$1/6$–$1/2$

Summary and Conclusion

Method

- Analytical comparison of domain-specific accuracy of the heuristics h^+, h^k, h^{PDB}, $h^{\mathrm{PDB}}_{\mathrm{add}}$.

Results

- Result: tradeoff between accuracy and computational cost.
  - h^+ mostly most accurate, but NP-hard to compute in general.
  - h^k and non-additive PDB heuristics get arbitrarily inaccurate.
  - Additive PDB heuristics
    - competitive with h^+ in accuracy,
    - easier to compute.
- Open: Other heuristics (additive h^k, ...), automatic optimization of pattern collections for additive PDB heuristics.

Thank you for your attention!