

# Minimizing Necessary Observations for Nondeterministic Planning

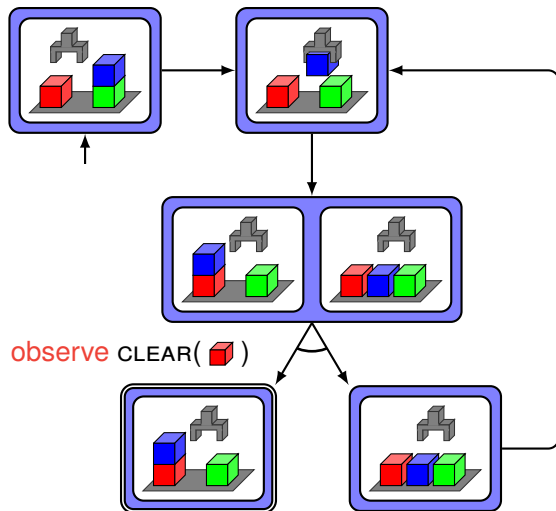
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# Motivation

Example: Partially Observable BLOCKSWORLD Domain with  
Nondeterministic PUT-DOWN Operator



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# Motivation

Example: Partially Observable BLOCKSWORLD Domain with  
Nondeterministic PUT-DOWN Operator

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Observing CLEAR predicate sufficient to find solutions.

Initial state known  $\Rightarrow$  overhead camera sufficient as a sensor.

**Question:** How to find minimal sets of variables sufficient for solution existence in arbitrary POND planning tasks?

**Remark:** Here, “solution” means **strong cyclic plan**:

- **closed:** defined for all belief states it may reach, and
- **proper:** no dead ends

## Problem OBSERVEINCLMIN:

- **Input:** POND planning task  $\Pi = \langle \mathcal{V}, B_0, B_\star, \mathcal{A}, \mathcal{W} \rangle$  with
  - state variables  $\mathcal{V}$
  - initial belief state  $B_0$
  - goal description  $B_\star$
  - nondeterministic actions  $\mathcal{A}$
  - possibly observable variables  $\mathcal{W} \subseteq \mathcal{V}$
- **Output:** Inclusion-minimal set of variables  $\mathcal{O} \subseteq \mathcal{W}$  such that there exists a strong cyclic plan for  $\Pi$  observing only variables from  $\mathcal{O}$ , or NONE if no such set  $\mathcal{O}$  exists.

### Theorem (Rintanen, 2004)

*The strong cyclic plan existence problem for POND planning, PLANEXPOND, is 2-EXPTIME-complete.* □

### Theorem

*OBSERVEINCLMIN is 2-EXPTIME-complete.*

### Proof.

- Trivial reduction from PLANEXPOND  
⇒ OBSERVEINCLMIN 2-EXPTIME-hard.
- Naive algorithm iterating over all subsets of  $\mathcal{W}$   
⇒ OBSERVEINCLMIN  $\in$  2-EXPTIME. □

**Question:** Can we improve over the naive algorithm from the proof?

**Assumption:** No obviously irrelevant variables in  $\mathcal{W}$ .  
Ignore variables known in  $B_0$  and never made unknown by any action.

# Baseline Algorithm

## Simple Greedy Algorithm

```
function SIMPLEGREEDYSEARCH( $\Pi$ ):  
  if  $\Pi$  is unsolvable then  
    return NONE  
  Compute some plan  $\pi$  for  $\Pi$   
  Let  $\mathcal{O}$  be the set of variables actually observed in  $\pi$   
  while  $\Pi$  still solvable with some  $o$  removed from  $\mathcal{O}$  do  
    Remove  $o$  from  $\mathcal{O}$   
  return  $\mathcal{O}$ 
```

### Theorem

*Function* SIMPLEGREEDYSEARCH

- runs in 2-EXPTIME,
- correctly solves OBSERVEINCLMIN. □

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# Plan Reuse

## Motivation

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- ✓ SIMPLEGREEDYSEARCH correct
- ✓ SIMPLEGREEDYSEARCH asymptotically optimal
- ✗ SIMPLEGREEDYSEARCH naive and inefficient
  - ▶ Look for ways to speed up SIMPLEGREEDYSEARCH!
  - ▶ Reuse portions of plan not affected by dropping a variable.

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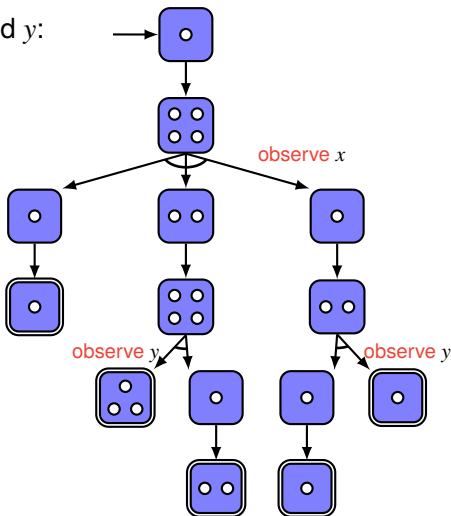
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# Plan Reuse

Example,  $\mathcal{W} = \{x, y\}$

Plan observing  $x$  and  $y$ :



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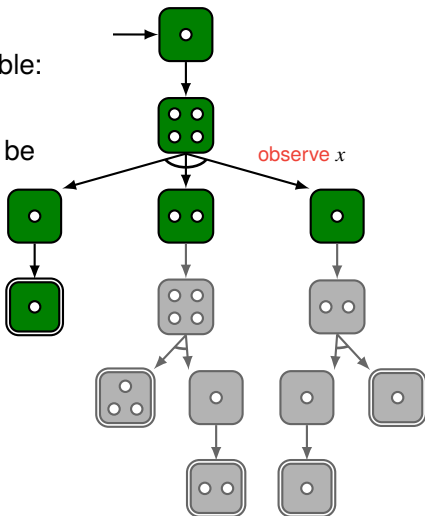
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# Plan Reuse

Example,  $\mathcal{W} = \{x, y\}$

Reusable plan fragment if  $y$  unobservable:

- $y$  known or
- never needs to be observed before goal



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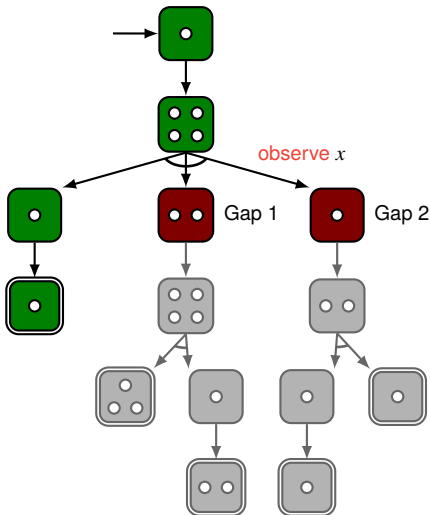
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# Plan Reuse

Example,  $\mathcal{W} = \{x, y\}$

Gap states that need replanning



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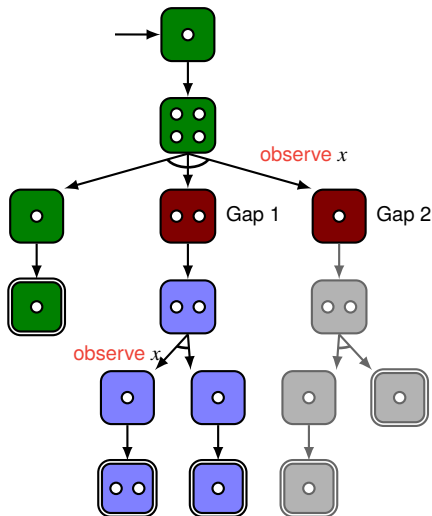
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# Plan Reuse

Example,  $\mathcal{W} = \{x, y\}$

Plan to fill gap 1



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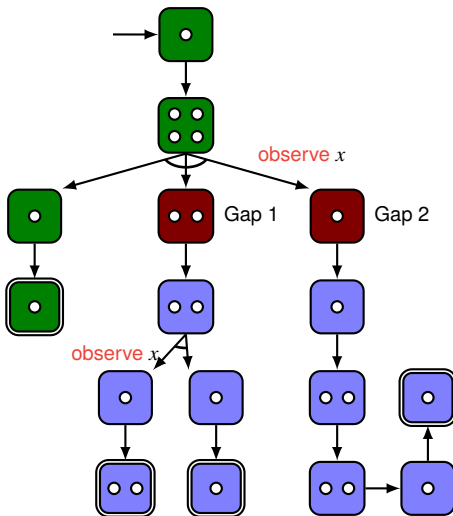
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# Plan Reuse

Example,  $\mathcal{W} = \{x, y\}$

Plan to fill gap 2



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# Plan Reuse

## Soundness

### To eliminate variable $y$ :

- Let  $\pi$  be the old plan still observing  $y$ .
- Identify gaps  $1, \dots, n$  in  $\pi$ .
- Let  $\pi_y$  be the reusable fragment of  $\pi$ .
- Let  $\pi_j, j = 1, \dots, n$ , be the new sub-plans filling the gaps.
- Let  $\pi' = \pi_y \oplus \pi_1 \oplus \dots \oplus \pi_n$  ( $\oplus =$  function overriding).

### Lemma

*If  $\pi$  and all  $\pi_j$  are strong cyclic plans, then so is  $\pi'$ .*

### Proof sketch.

In  $\pi'$ , “last” subplan “wins”. Thus, closedness and properness of  $\pi$  and all  $\pi_j$  carry over to  $\pi'$ . □

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To minimize set of observation variables:

- Eliminate variables one by one, if possible.

### Theorem

*Plan  $\pi$  resulting from successive elimination of variables is strong cyclic plan.*

### Proof sketch.

Base case + inductive application of previous lemma. □

### Remarks:

- In induction, skip gaps filled/circumvented by chance when filling earlier gap.
- In elimination step, existence of  $\pi_j$  not guaranteed. Resulting  $\pi'$  not necessarily with inclusion minimal set  $\mathcal{O}$ .

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### Remark:

Observation sets found with plan reuse can be suboptimal.

**Example:** Let  $\Pi$  with

- Propositional variables  $\mathcal{V} = \{a, b, c\}$ ,
- Initial belief state  $B_0 = \bar{a}\bar{b}\bar{c}$ ,
- Goal belief state  $B_\star = c$ ,
- Observable variables  $\mathcal{W} = \{b\}$ ,
- Actions  $\mathcal{A} = \{a_1, a_2, a_3, a_4\}$ , where
  - $a_1 = \langle \bar{a} \rightarrow a \rangle$ ,
  - $a_2 = \langle \bar{b} \rightarrow (b \text{ or } \top) \rangle$ ,
  - $a_3 = \langle b \rightarrow c \rangle$ ,
  - $a_4 = \langle \bar{a}\bar{b}\bar{c} \rightarrow c \rangle$ .

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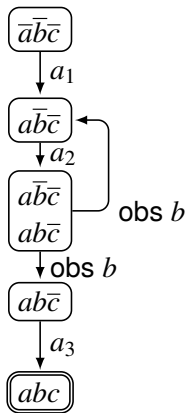
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# Plan Reuse

## Suboptimality

Example (ctd.): Possible plan for  $\Pi$ :



With observation of  $b$ .

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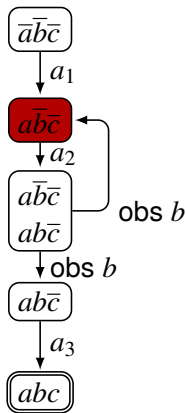
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# Plan Reuse

## Suboptimality

Example (ctd.): Possible plan for  $\Pi$ :



With observation of  $b$ .

Only **gap state**.

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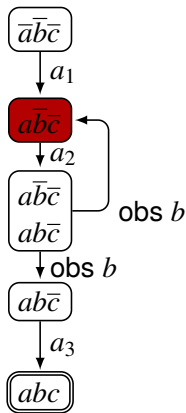
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# Plan Reuse

## Suboptimality

Example (ctd.): Possible plan for  $\Pi$ :



With observation of  $b$ .

No plan for  $a\bar{b}\bar{c}$  without observing  $b$ :

- only applicable action  $a_2$  makes  $b$  unknown
- then all actions inapplicable

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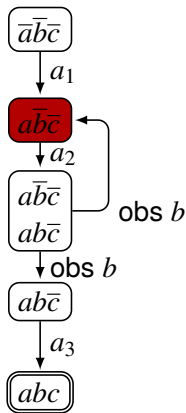
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# Plan Reuse

## Suboptimality

Example (ctd.): Possible plan for  $\Pi$ :



With observation of  $b$ .

No plan for  $a\bar{b}\bar{c}$  without observing  $b$ :

- only applicable action  $a_2$  makes  $b$  unknown
- then all actions inapplicable

Variable  $b$  cannot be removed  
 $\Rightarrow$  solution  $\mathcal{O} = \{b\}$

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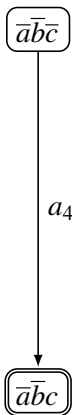
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# Plan Reuse

## Suboptimality

Example (ctd.): Possible plan for  $\Pi$ :



Plan for  $\bar{a}\bar{b}\bar{c}$  without observation of  $b$  exists.

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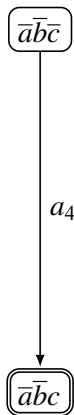
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# Plan Reuse

## Suboptimality

Example (ctd.): Possible plan for  $\Pi$ :



Plan for  $\bar{a}\bar{b}\bar{c}$  without  
observation of  $b$  exists.

$\Rightarrow$  optimal solution  $\mathcal{O}^* = \emptyset$

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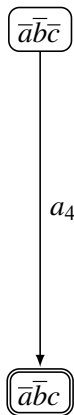
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# Plan Reuse

## Suboptimality

Example (ctd.): Possible plan for  $\Pi$ :



Plan for  $\bar{a}\bar{b}\bar{c}$  without observation of  $b$  exists.

$\Rightarrow$  optimal solution  $\mathcal{O}^* = \emptyset$

$\Rightarrow$  solution  $\mathcal{O} = \{b\}$  found with plan reuse was suboptimal

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# Functional Dependencies

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## Further enhancement: functional dependencies

- **Idea:** If value of variable  $o$  can be inferred from values of observed variables  $o^1, \dots, o^n$ , need not observe  $o$ .
- Identify such functional dependencies in plan  $\pi$ .
- Replace observations of  $o$  in  $\pi$  by observation of  $o^1, \dots, o^n$ .
- **Remark:** functional dependencies only have to hold in states reachable following  $\pi$ , not necessarily in all reachable states.
- **Implemented:** only exactly-one mutexes between propositional variables.

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# Empirical Evaluation

## Runtimes

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- Implementation on top of MYND planner<sup>1</sup>.
- LAO\* search [Hansen & Zilberstein, 2001] guided by FF heuristic [Hoffmann & Nebel, 2001].
- Domains:
  - BLOCKSWORLD
  - FIRSTRESPONDERS
  - TIDYUP
- Legend:
  - Gr = greedy
  - PR = plan reuse
  - FD = functional dependencies

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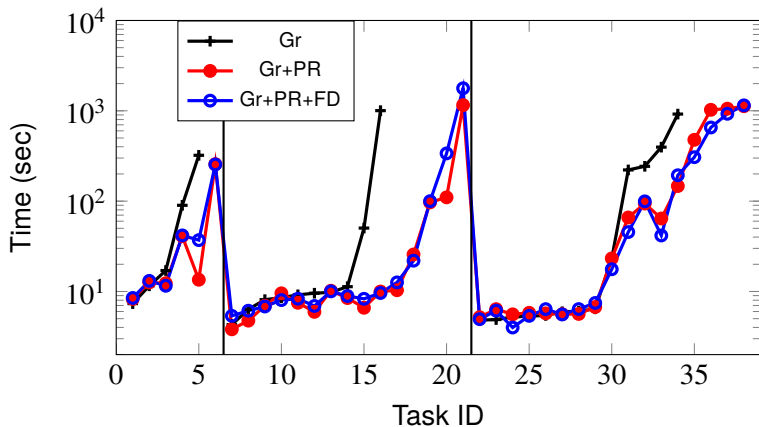
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<sup>1</sup><https://bitbucket.org/robertmattmueller/mynd>

# Empirical Evaluation

## Runtimes

Overall runtime needed for finding final observation set.



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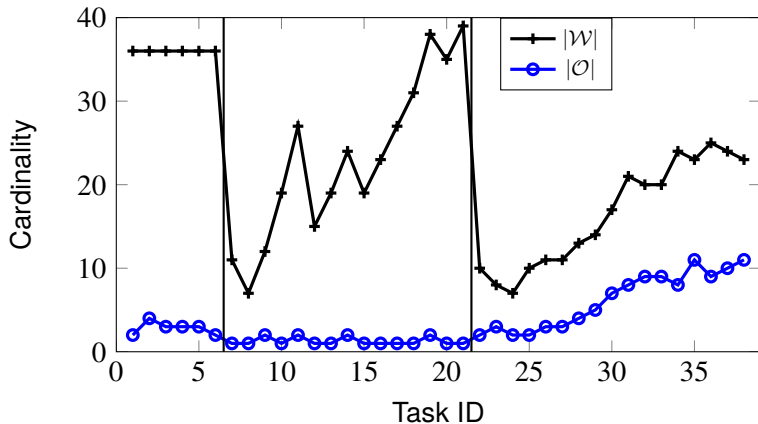
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# Empirical Evaluation

## Observation Set Cardinalities

Cardinalities of the observation sets before/after minimization.



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# Empirical Evaluation

## Domain-specific Observations

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### Variables in resulting observation sets:

- **BLOCKSWORLD**: mostly

- **ONTABLE**

- **CLEAR**

(either of them alone is sufficient.)

- **FIRSTRESPONDERS**:

- **FIRE** (in all tasks, for relevant locations)

- In one instance without road to hospital:

- **VICTIMSTATUS** – needs to be observed for applicability of **TREATVICTIMONSCENE**.

- **TIDYUP**: relevant instances of

- **GRIPPERSTATUS**

- **TABLECLEAN**

- **DOORSTATE**

- **ROBOTLOCATION**

- **CUPLOCATION**

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# Conclusion and Future Work

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## Conclusion:

- Theory: **OBSERVEINCLMIN** is **2-EXPTIME-complete**.
- Presented asymptotically optimal baseline **greedy top-down algorithm** for **OBSERVEINCLMIN**.
- Extended it with
  - **plan reuse** (pays off) and
  - **functional dependencies** (do not really pay off).

## Future work:

- Complement top-down with **bottom-up procedure**.
- Investigate **variable ordering heuristics** for the iteration over candidate variables for removal.
- Study problem on **domain** instead of planning task **level**.

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