Abstractions for Planning with State-Dependent Action Costs

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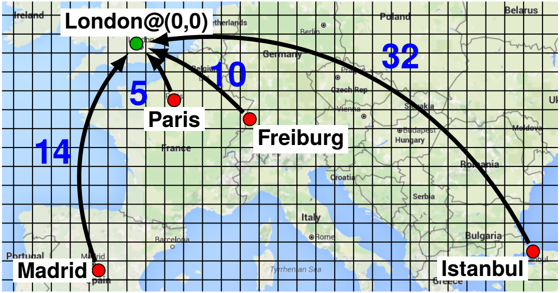
What are State-Dependent Action Costs?



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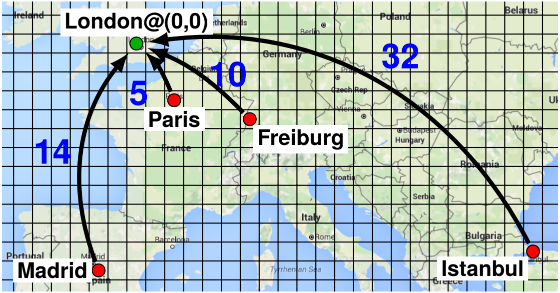
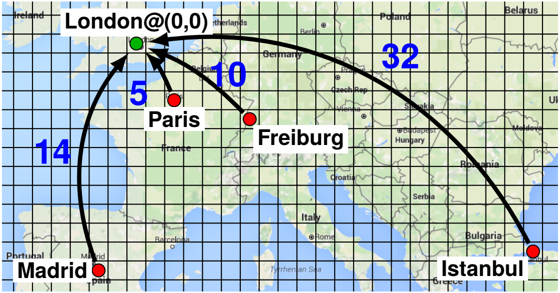


Action costs: unit — constant — state-dependent

$$\textit{cost}(\textit{fly}(Madrid, London)) = 1, \quad \textit{cost}(\textit{fly}(Paris, London)) = 1,$$
$$\textit{cost}(\textit{fly}(Freiburg, London)) = 1, \quad \textit{cost}(\textit{fly}(Istanbul, London)) = 1.$$

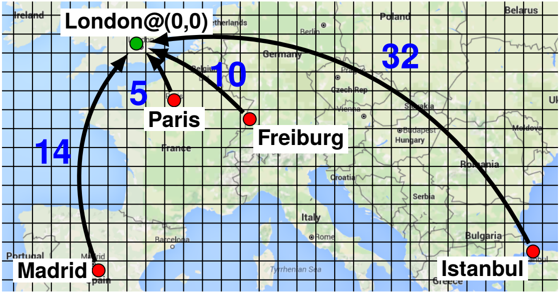
What are State-Dependent Action Costs?



Action costs: unit — constant — state-dependent

$$cost(fly(Madrid, London)) = 14, \quad cost(fly(Paris, London)) = 5,$$
$$cost(fly(Freiburg, London)) = 10, \quad cost(fly(Istanbul, London)) = 32.$$

What are State-Dependent Action Costs?



Action costs: unit — constant — state-dependent

$$cost(flyTo(London)) = |x_{\text{London}} - x_{\text{current}}| + |y_{\text{London}} - y_{\text{current}}|$$
$$= |x_{\text{current}}| + |y_{\text{current}}|.$$

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Why Study State-Dependent Action Costs?

Advantages:

- Structured and “natural”
- Exponentially more compact, fewer redundancies
- Relevant to applications

$\leadsto$ benefits for:

- Human modelers
- Computers/algorithms (exploit structure!)

Handling State-Dependent Action Costs

State of the art:

- Different compilations to constant-cost tasks
- Generalized additive heuristic h^{add}
- Generalized relaxed planning graph to compute h^{add}

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Open questions:

- Optimal planning with state-dependent costs.
- $\rightsquigarrow$ admissible abstraction heuristics
  - abstract transition costs (always/sometimes) efficiently computable?
  - empirical performance?

Edge-Valued Multi-Valued Decision Diagrams

Appropriate data structure to represent action cost functions:

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Edge-Valued Multi-Valued Decision Diagrams (EVMDDs)

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Reasons:

- Follow naturally from desired properties of compilations
- Exhibit additive structure
- Attribute partial costs to facts responsible for them
- Often compact

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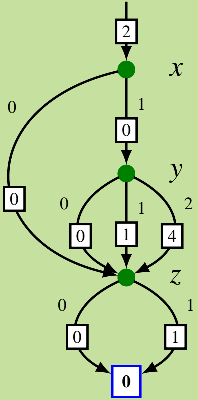
$\rightsquigarrow$ try to exploit additive structure exhibited by them!

Edge-Valued Multi-Valued Decision Diagrams

Example (EVMDD Evaluation)

$cost_o = xy^2 + z + 2$ $\qquad \mathcal{D}_x = \mathcal{D}_z = \{0,1\},\ \mathcal{D}_y = \{0,1,2\}$



- Directed acyclic graph
- Dangling incoming edge
- Single terminal node **0**
- Decision nodes with:
  - decision variables
  - edge label
  - edge weights

Edge-Valued Multi-Valued Decision Diagrams

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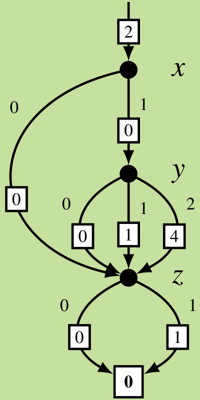

- $s = \{x \mapsto 1,\ y \mapsto 2,\ z \mapsto 0\}$
- $\mathit{cost}_o(s) =$

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Edge-Valued Multi-Valued Decision Diagrams

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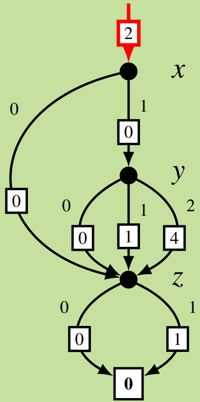

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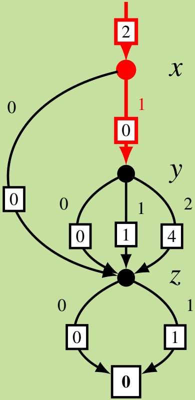

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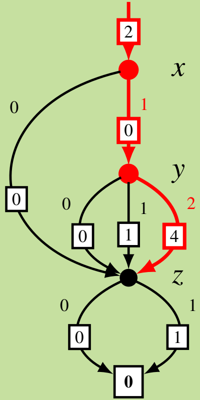

- $s = \{x \mapsto 1,\ y \mapsto 2,\ z \mapsto 0\}$
- $\mathit{cost}_o(s) = 2 + 0 + 4 +$

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Edge-Valued Multi-Valued Decision Diagrams

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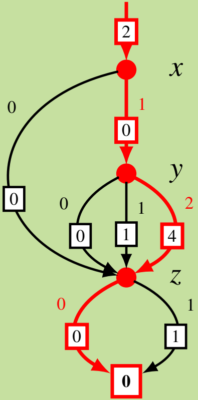


- $s = \{x \mapsto 1,\ y \mapsto 2,\ z \mapsto 0\}$
- $\mathit{cost}_o(s) = 2 + 0 + 4 + 0 = 6$

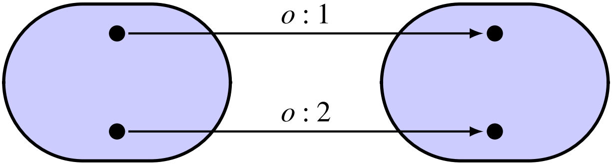
Edge-Valued Multi-Valued Decision Diagrams

Properties of EVMDDs:

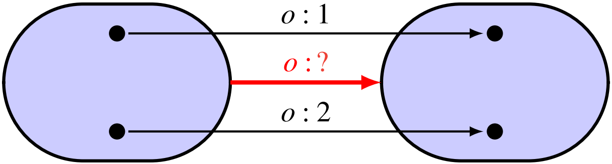
- ✔ Existence
- ✔ Uniqueness/canonicity (if reduced and ordered)
- ✔ Basic arithmetic operations supported

(Lai et al., 1996; Ciardo and Siminiceanu, 2002)

Abstraction Heuristics



Abstraction Heuristics



Question: What are the abstract action costs?

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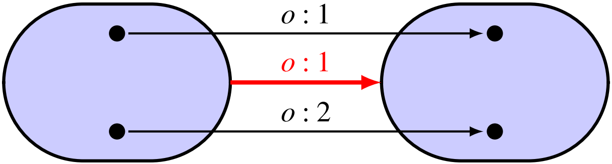
Abstraction Heuristics

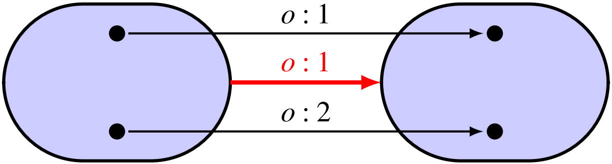


Question: What are the abstract action costs?

Answer: For admissibility, in abstract state s^{abs}, operator o should cost

$$\mathit{cost}_o(s^{abs}) = \min_{\substack{\text{concrete state } s \\ \text{abstracted to } s^{abs}}} \mathit{cost}_o(s).$$

Abstraction Heuristics



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$$cost_o(s^{abs}) = \min_{\substack{\text{concrete state } s \\ \text{abstracted to } s^{abs}}} cost_o(s).$$

Problem: exponentially many states to minimize over

Aim: Compute $cost_o(s^{abs})$ efficiently (given EVMDD for $cost_o(s)$).

Cartesian Abstractions

We will see: this is possible if the abstraction is Cartesian or coarser.

(This includes projections and domain abstractions.)

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Definition (Cartesian abstraction (Seipp and Helmert, 2013))

A set of states s^{abs} is Cartesian if it is of the form

$$D_1 \times \cdots \times D_n,$$

where $D_i \subseteq \mathcal{D}_i$ for all $i = 1, \ldots, n$.

An abstraction is Cartesian if all its abstract states are Cartesian sets.

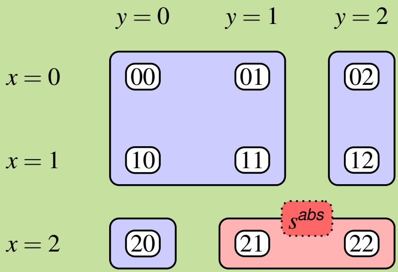
Intuition: In s^{abs}, variables are abstracted independently.

$\rightsquigarrow$ exploit independence when computing abstract costs.

Cartesian Abstractions

Example (Cartesian abstraction)

Some Cartesian abstraction over x, y



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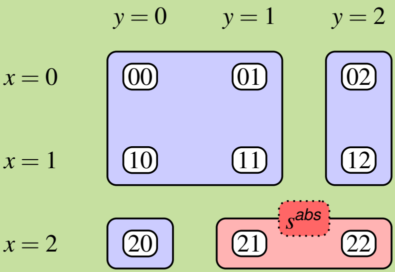
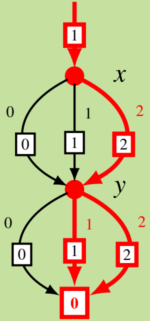


Cost $x+y+1$

(edges consistent with s^{abs})



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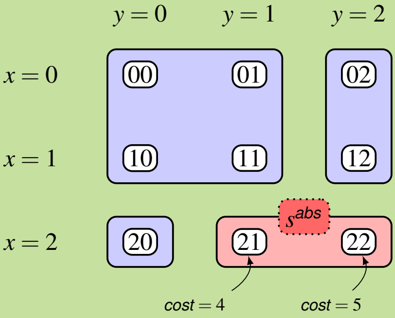

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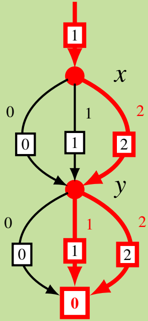

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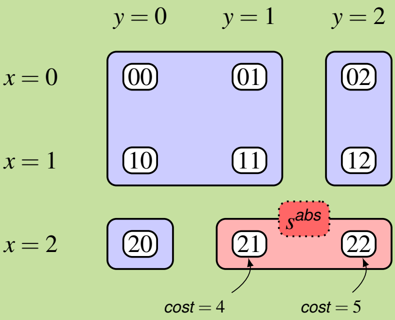

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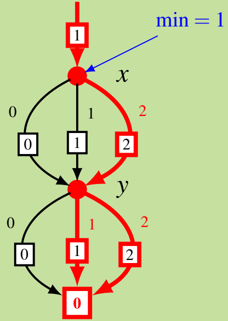

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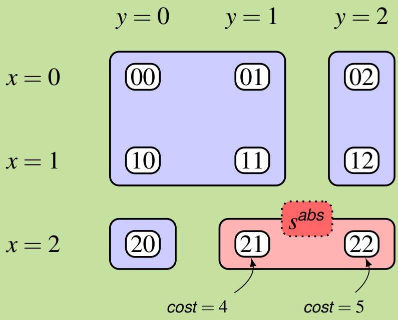

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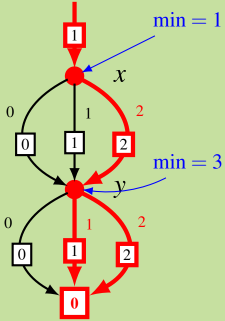

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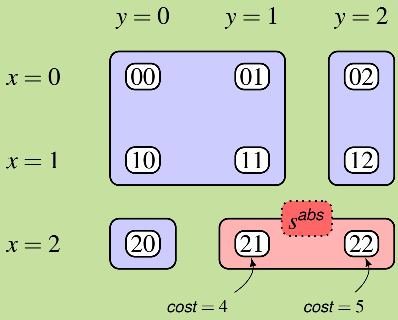

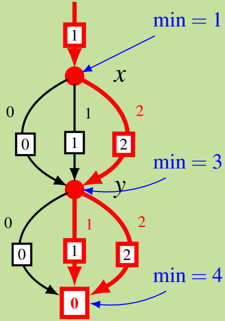

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Cartesian Abstractions

What happens here? *or:*

Why does the topsort EVMDD traversal correctly compute $cost_o(s^{abs})$?

1. For each Cartesian state s^{abs} and each variable x, each value $d \in \mathcal{D}_x$ is either consistent with s^{abs} or not.
2. This implies: at all decision nodes associated with variable x, some outgoing edges are enabled, others are disabled.

   This is independent from all other decision nodes/variables.
3. This allows local minimizations over (linearly many) edges instead of global minimization over (exponentially many) paths in the EVMDD when computing minimum costs.

$\rightsquigarrow$ polynomial in EVMDD size!

Cartesian Abstractions

Not Cartesian!

If abstraction is not Cartesian: two variables can be

- independent in the cost function ($\rightsquigarrow$ compact EVMDD), but
- dependent in the abstraction.

$\rightsquigarrow$ cannot consider independent parts of the EVMDD separately.

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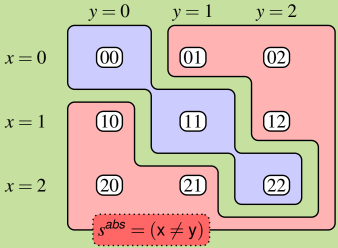
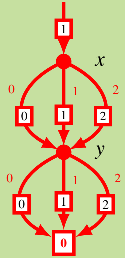
Example (Non-Cartesian abstraction)

cost : $x + y + 1$, *cost*$(s^{abs}) = 2$, local minimization: $1 \rightsquigarrow$ underestimate!

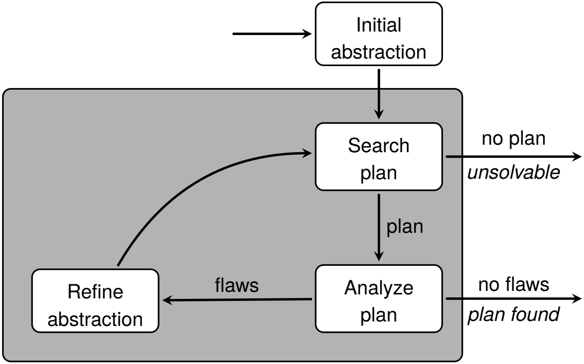




Counterexample-Guided Abstraction Refinement

Wanted: principled way of computing Cartesian abstractions.

⇝ Counterexample-Guided Abstraction Refinement (CEGAR)



Possible flaws in abstract plan:

1. Concrete state does not fit abstract state
   (concrete and abstract traces diverge)
2. Operator not applicable in concrete state
3. Trace completed, but goal not reached

Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

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Here, we need to consider a further type of flaw:

4. Cost-mismatch flaw: Action more costly in concrete state than in abstract state

Counterexample-Guided Abstraction Refinement

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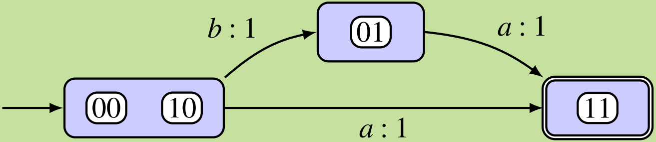
$\rightsquigarrow$ resolve cost-mismatch flaws with additional refinement.

Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

Example (Cost-mismatch flaw)



$$a = \langle \top, \quad x \wedge y \rangle, \quad \mathit{cost}_a = 2x + 1 \qquad s_0 = 10$$

$$b = \langle \top, \neg x \wedge y \rangle, \quad \mathit{cost}_b = 1 \qquad s_\star = x \wedge y$$

Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

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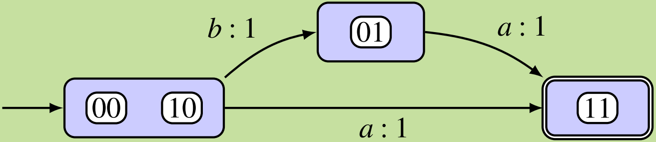

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- Optimal abstract plan: $\langle a \rangle$ (abstract cost 1)

Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

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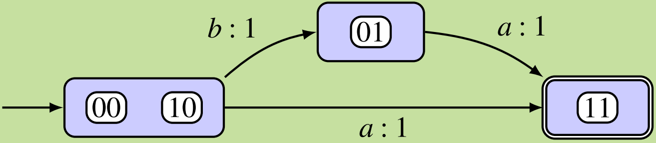

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Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

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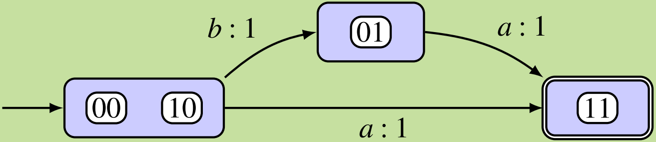

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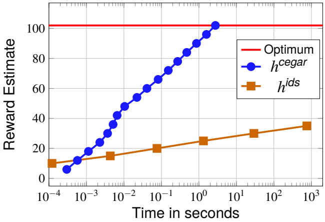
- Optimal abstract plan: $\langle a \rangle$ (abstract cost 1)
- This is also a concrete plan (concrete cost 3)
- But optimal concrete plan: $\langle b, a \rangle$ (concrete and abstract cost 2)

Empirical Evaluation

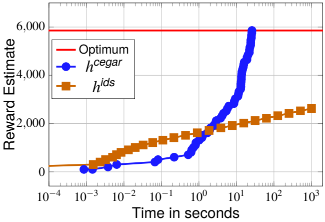
Experiment 1: Compare Anytime Behaviour of h^{cegar} and h^{ids}

Setting: IPPC benchmarks, Prost planner

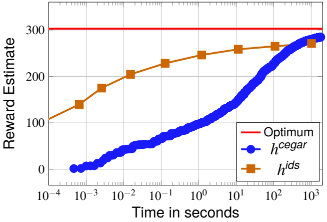
Academic Advising



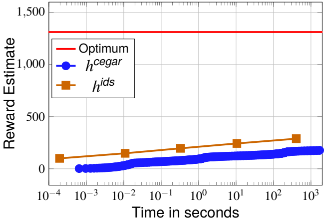
Triangle Tireworld



Tamarisk



Sysadmin



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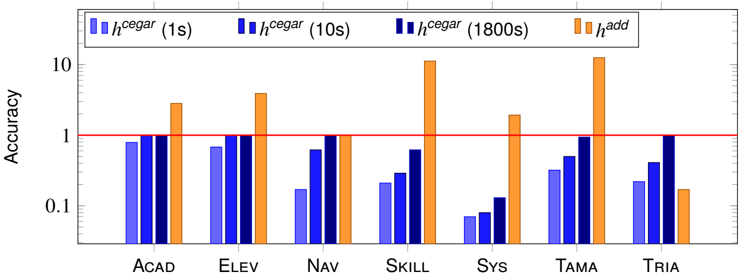
Empirical Evaluation

Experiment 2: Compare Accuracy of h^{cegar} and h^{add}

Observation/Question: h^{add} neither admissible nor anytime, but possibly more accurate than h^{cegar}? Let's see …



Conclusion:

- h^{cegar} never overestimates.
- h^{cegar} becomes more accurate over time.
- After sufficient time, accuracy of h^{cegar} comparable to that of h^{add}.

Summary

Our motivating challenges were:

- Understand when abstract costs are efficiently computable.
  - ✔ largely understood: if (and only if) abstraction is Cartesian
- Make abstraction heuristics state-dependent-action-cost aware.
  - ✔ done: defined/generalized
    - Cartesian abstractions
    - local EVMDD evaluation
    - generalized CEGAR
- Perform optimal planning with state-dependent action costs.
  - ✔ done: promising empirical results