Abstractions for Planning with State-Dependent Action Costs

Florian Geißer   Thomas Keller   Robert Mattmüller
June 15, 2016
ICAPS 2016, London, UK
What are State-Dependent Action Costs?

Action costs:
- Unit constant
- State-dependent cost

\[
\text{cost}(\text{flyTo}(\text{London})) = |x_{\text{London}} - x_{\text{current}}| + |y_{\text{London}} - y_{\text{current}}| = |x_{\text{current}}| + |y_{\text{current}}|.
\]
What are State-Dependent Action Costs?

Action costs: [unit] [constant] [state-dependent]

\[\text{cost} (\text{fly}(\text{Madrid}, \text{London})) = 1, \quad \text{cost} (\text{fly}(\text{Paris}, \text{London})) = 1, \]
\[\text{cost} (\text{fly}(\text{Freiburg}, \text{London})) = 1, \quad \text{cost} (\text{fly}(\text{Istanbul}, \text{London})) = 1.\]
What are State-Dependent Action Costs?

Action costs:  
- **unit**
- **constant**
- **state-dependent**

\[
\begin{align*}
\text{cost}(\text{fly}(\text{Madrid}, \text{London})) &= 14, \\
\text{cost}(\text{fly}(\text{Paris}, \text{London})) &= 5, \\
\text{cost}(\text{fly}(\text{Freiburg}, \text{London})) &= 10, \\
\text{cost}(\text{fly}(\text{Istanbul}, \text{London})) &= 32.
\end{align*}
\]
What are State-Dependent Action Costs?

Action costs:  

- **unit**
- **constant**
- **state-dependent**

**cost**(flyTo(London)) = |x_{London} − x_{current}| + |y_{London} − y_{current}|

= |x_{current}| + |y_{current}|.
Why Study State-Dependent Action Costs?

Advantages:
- Structured and “natural”
- Exponentially more compact, fewer redundancies
- Relevant to applications

〜〜 benefits for:
- Human modelers
- Computers/algorithms (exploit structure!)
Handling State-Dependent Action Costs

State of the art:

- Different compilations to constant-cost tasks
- Generalized additive heuristic $h^{add}$
- Generalized relaxed planning graph to compute $h^{add}$
Handling State-Dependent Action Costs

State of the art:
- Different compilations to constant-cost tasks
- Generalized additive heuristic $h^{add}$
- Generalized relaxed planning graph to compute $h^{add}$

Open questions:
- Optimal planning with state-dependent costs.
- Admissible abstraction heuristics
  - abstract transition costs (always/sometimes) efficiently computable?
  - empirical performance?
Edge-Valued Multi-Valued Decision Diagrams

Appropriate data structure to represent action cost functions:
Edge-Valued Multi-Valued Decision Diagrams

Appropriate *data structure to represent* action cost functions:

Edge-Valued Multi-Valued Decision Diagrams (EVMDDs)
Appropriate data structure to represent action cost functions:

Edge-Valued Multi-Valued Decision Diagrams (EVMDDs)

Reasons:

- Follow naturally from desired properties of compilations
- Exhibit additive structure
- Attribute partial costs to facts responsible for them
- Often compact
Edge-Valued Multi-Valued Decision Diagrams

Appropriate data structure to represent action cost functions:

Edge-Valued Multi-Valued Decision Diagrams (EVMDDs)

Reasons:

- Follow naturally from desired properties of compilations
- Exhibit additive structure
- Attribute partial costs to facts responsible for them
- Often compact

⇝ try to exploit additive structure exhibited by them!
Edge-Valued Multi-Valued Decision Diagrams

Example (EVMDD Evaluation)

\[ cost_o = xy^2 + z + 2 \]

\[ D_x = D_z = \{0, 1\}, \quad D_y = \{0, 1, 2\} \]

- Directed acyclic graph
- Dangling incoming edge
- Single terminal node \(0\)
- Decision nodes with:
  - decision variables
  - edge label
  - edge weights
Edge-Valued Multi-Valued Decision Diagrams

Example (EVMDD Evaluation)

\[ \text{cost}_o = xy^2 + z + 2 \]

\[ D_x = D_z = \{0, 1\}, \quad D_y = \{0, 1, 2\} \]

\[ s = \{x \mapsto 1, \; y \mapsto 2, \; z \mapsto 0\} \]

\[ \text{cost}_o(s) = \]
Example (EVMDD Evaluation)

\[ cost_o = xy^2 + z + 2 \]

\[ D_x = D_z = \{0, 1\}, \ D_y = \{0, 1, 2\} \]

\[ s = \{x \mapsto 1, y \mapsto 2, z \mapsto 0\} \]

\[ cost_o(s) = 2 + \]
Edge-Valued Multi-Valued Decision Diagrams

**Example (EVMDD Evaluation)**

\[ \text{cost}_o = xy^2 + z + 2 \]

\[ D_x = D_z = \{0, 1\}, \quad D_y = \{0, 1, 2\} \]

\[ s = \{x \mapsto 1, \ y \mapsto 2, \ z \mapsto 0\} \]

\[ \text{cost}_o(s) = 2 + 0 + \]

\[ \text{cost}_o(s) = 2 + 0 + \]
Edge-Valued Multi-Valued Decision Diagrams

Example (EVMDD Evaluation)

\[ \text{cost}_o = xy^2 + z + 2 \]

\[ D_x = D_z = \{0, 1\}, \quad D_y = \{0, 1, 2\} \]

\[ s = \{x \mapsto 1, \ y \mapsto 2, \ z \mapsto 0\} \]

\[ \text{cost}_o(s) = 2 + 0 + 4 + \]
Edge-Valued Multi-Valued Decision Diagrams

Example (EVMDD Evaluation)

\[ \text{cost}_o = xy^2 + z + 2 \]
\[ D_x = D_z = \{0, 1\}, \quad D_y = \{0, 1, 2\} \]

\[ s = \{x \mapsto 1, \; y \mapsto 2, \; z \mapsto 0\} \]
\[ \text{cost}_o(s) = 2 + 0 + 4 + 0 = 6 \]
Properties of EVMDDs:

- Existence
- Uniqueness/canonicity (if reduced and ordered)
- Basic arithmetic operations supported

(Lai et al., 1996; Ciardo and Siminiceanu, 2002)
Abstraction Heuristics

Question: What are the abstract action costs?
Answer: For admissibility, in abstract state $s_{abs}$, operator $o$ should cost $\text{cost}_o(s_{abs}) = \min \text{concrete state } s \text{abstracted to } s_{abs}$.

Problem: exponentially many states to minimize over
Aim: Compute $\text{cost}_o(s_{abs})$ efficiently (given EVMDD for $\text{cost}_o(s)$)

June 15, 2016 Geißer, Keller, Mattmüller – Abstractions for Planning with State-Dependent Action Costs
Abstraction Heuristics

Question: What are the abstract action costs?

For admissibility, in abstract state $s_{abs}$, operator $o$ should cost $\text{cost}_o(s_{abs}) = \min_{\text{concrete state } s \text{ abstracted to } s_{abs}} \text{cost}_o(s)$.

Problem: exponentially many states to minimize over

Aim: Compute $\text{cost}_o(s_{abs})$ efficiently (given EVMDD for $\text{cost}_o(s)$).
**Question:** What are the abstract action costs?

**Answer:** For admissibility, in abstract state $s^{\text{abs}}$, operator $o$ should cost

\[
\text{cost}_o(s^{\text{abs}}) = \min_{\text{concrete state } s \text{ abstracted to } s^{\text{abs}}} \text{cost}_o(s).
\]
Abstraction Heuristics

Question: What are the abstract action costs?

Answer: For admissibility, in abstract state $s^{abs}$, operator $o$ should cost

$$cost_o(s^{abs}) = \min_{\text{concrete state } s \text{ abstracted to } s^{abs}} cost_o(s).$$

Problem: exponentially many states to minimize over

Aim: Compute $cost_o(s^{abs})$ efficiently (given EVMDD for $cost_o(s)$).
Cartesian Abstractions

We will see: this is possible if the abstraction is Cartesian or coarser.

(This includes projections and domain abstractions.)
**Cartesian Abstractions**

We will see: this is possible if the abstraction is **Cartesian** or coarser.
(This includes projections and domain abstractions.)

**Definition (Cartesian abstraction (Seipp and Helmert, 2013))**

A set of states $s^{abs}$ is Cartesian if it is of the form

$$D_1 \times \cdots \times D_n,$$

where $D_i \subseteq D_i$ for all $i = 1, \ldots, n$.

An abstraction is Cartesian if all its abstract states are Cartesian sets.

**Intuition:** In $s^{abs}$, variables are abstracted independently.
$\rightsquigarrow$ exploit independence when computing abstract costs.
Cartesian Abstractions

Example (Cartesian abstraction)

Some Cartesian abstraction over \( x, y \)

\[
\begin{align*}
\text{Cost} & = x + y + 1 \\
\text{Cost} & = x + y + 1 \\
\text{Cost} & = x + y + 1 \\
\end{align*}
\]

[Diagram showing a 3x3 grid with states labeled by \( x, y \) and costs indicated]
Some Cartesian abstraction over $x, y$

Cost $x + y + 1$
(edges consistent with $s^{abs}$)
Cartesian Abstractions

Example (Cartesian abstraction)

Some Cartesian abstraction over $x$, $y$

Cost $x + y + 1$
(edges consistent with $s^{abs}$)

$$\begin{align*}
&x = 0 \\
&y = 0 \quad 00 \\
&y = 1 \quad 01 \\
&y = 2 \quad 02 \\
&x = 1 \\
&y = 0 \quad 10 \\
&y = 1 \quad 11 \\
&y = 2 \quad 12 \\
&x = 2 \\
&y = 0 \quad 20 \\
&y = 1 \quad 21 \\
&y = 2 \quad 22 \\
\end{align*}$$
Example (Cartesian abstraction)

Some Cartesian abstraction over $x$, $y$

Cost $x + y + 1$
(edges consistent with $s^{abs}$)

$\begin{align*}
  x = 0 & \quad y = 0 & \quad 00 \\
  x = 0 & \quad y = 1 & \quad 01 \\
  x = 0 & \quad y = 2 & \quad 02 \\
  x = 1 & \quad y = 0 & \quad 10 \\
  x = 1 & \quad y = 1 & \quad 11 \\
  x = 1 & \quad y = 2 & \quad 12 \\
  x = 2 & \quad y = 0 & \quad 20 \\
  x = 2 & \quad y = 1 & \quad 21 \\
  x = 2 & \quad y = 2 & \quad 22 \\
\end{align*}$

$cost = 4 \quad cost = 5$

$\min = 1$

$\begin{align*}
  x & \quad y \\
  0 & \quad 0 \\
  0 & \quad 1 \\
  0 & \quad 2 \\
  1 & \quad 0 \\
  1 & \quad 1 \\
  1 & \quad 2 \\
  2 & \quad 0 \\
  2 & \quad 1 \\
  2 & \quad 2 \\
\end{align*}$
Example (Cartesian abstraction)

Some Cartesian abstraction over $x, y$

Cost $x + y + 1$
(edges consistent with $s^{abs}$)

$x = 0$

<table>
<thead>
<tr>
<th></th>
<th>$y = 0$</th>
<th>$y = 1$</th>
<th>$y = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = 0$</td>
<td>00</td>
<td>01</td>
<td>02</td>
</tr>
<tr>
<td>$x = 1$</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>$x = 2$</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
</tbody>
</table>

$cost = 4$ $cost = 5$

$min = 1$ $min = 3$
Cartesian Abstractions

Example (Cartesian abstraction)

Some Cartesian abstraction over $x$, $y$

$$\begin{align*}
\text{Cost } x + y + 1 \\
(\text{edges consistent with } s^{\text{abs}})
\end{align*}$$

$\begin{align*}
x = 0 & \\
00 & \quad 01 & \quad 02 \\
\text{cost} = 4 \\
x = 1 & \\
10 & \quad 11 & \quad 12 \\
x = 2 & \\
20 & \quad 21 & \quad 22 \\
\text{cost} = 5 \\
\end{align*}$

$\begin{align*}
\text{min} = 1 \\
\text{min} = 3 \\
\text{min} = 4 \\
\end{align*}$
Cartesian Abstractions

What happens here?  

or:

Why does the topsort EVMDD traversal correctly compute $\text{cost}_o(s^{abs})$?

1. For each Cartesian state $s^{abs}$ and each variable $x$, each value $d \in D_x$ is either consistent with $s^{abs}$ or not.

2. This implies: at all decision nodes associated with variable $x$, some outgoing edges are enabled, others are disabled. This is independent from all other decision nodes/variables.

3. This allows local minimizations over (linearly many) edges instead of global minimization over (exponentially many) paths in the EVMDD when computing minimum costs.

$\rightsquigarrow$ polynomial in EVMDD size!
If abstraction is not Cartesian: two variables can be
- **independent** in the cost function (\(\leadsto\) compact EVMDD), but
- **dependent** in the abstraction.

\(\leadsto\) cannot consider independent parts of the EVMDD separately.
Cartesian Abstractions
Not Cartesian!

If abstraction is not Cartesian: two variables can be
- independent in the cost function (\(\rightarrow\) compact EVMDD), but
- dependent in the abstraction.
\(\rightarrow\) cannot consider independent parts of the EVMDD separately.

Example (Non-Cartesian abstraction)

\(cost: x + y + 1, \ cost(s^{abs}) = 2\), local minimization: 1 \(\rightarrow\) underestimate!

\[s^{abs} = (x \neq y)\]
**Counterexample-Guided Abstraction Refinement**

**Wanted:** principled way of computing Cartesian abstractions.

∽∽ Counterexample-Guided Abstraction Refinement (CEGAR)
Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

Possible flaws in abstract plan:

1. Concrete state does not fit abstract state (concrete and abstract traces diverge)
2. Operator not applicable in concrete state
3. Trace completed, but goal not reached

⇝ resolve cost-mismatch flaws with additional refinement.
Possible flaws in abstract plan:

1. Concrete state does not fit abstract state (concrete and abstract traces diverge)
2. Operator not applicable in concrete state
3. Trace completed, but goal not reached

Here, we need to consider a further type of flaw:

4. Cost-mismatch flaw: Action more costly in concrete state than in abstract state
Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

Possible flaws in abstract plan:

1. Concrete state does not fit abstract state (concrete and abstract traces diverge)
2. Operator not applicable in concrete state
3. Trace completed, but goal not reached

Here, we need to consider a further type of flaw:

4. Cost-mismatch flaw: Action more costly in concrete state than in abstract state

\(\rightarrow\) resolve cost-mismatch flaws with additional refinement.
Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

Example (Cost-mismatch flaw)

\[
a = \langle \top, \ x \land y \rangle, \ \text{cost}_a = 2x + 1 \quad s_0 = 10
\]

\[
b = \langle \top, \ \neg x \land y \rangle, \ \text{cost}_b = 1 \quad s_* = x \land y
\]
Counterexample-Guided Abstraction Refinement

Cost-Mismatch Flaws

Example (Cost-mismatch flaw)

\[ a = \langle \top, x \wedge y \rangle, \quad \text{cost}_a = 2x + 1 \]
\[ b = \langle \top, \neg x \wedge y \rangle, \quad \text{cost}_b = 1 \]
\[ s_0 = 10 \quad s_* = x \wedge y \]

- Optimal abstract plan: \( \langle a \rangle \) (abstract cost 1)
Counterexample-Guided Abstraction Refinement
Cost-Mismatch Flaws

Example (Cost-mismatch flaw)

\[ a = \langle \top, x \land y \rangle, \quad \text{cost}_a = 2x + 1 \]
\[ b = \langle \top, \neg x \land y \rangle, \quad \text{cost}_b = 1 \]
\[ s_0 = 10 \]
\[ s_* = x \land y \]

- Optimal abstract plan: \( \langle a \rangle \) (abstract cost 1)
- This is also a concrete plan (concrete cost 3)
Example (Cost-mismatch flaw)

\[ a = \langle \top, \, x \land y \rangle, \quad \text{cost}_a = 2x + 1 \quad \text{and} \quad s_0 = 10 \]

\[ b = \langle \top, \, \neg x \land y \rangle, \quad \text{cost}_b = 1 \quad \text{and} \quad s_* = x \land y \]

- Optimal abstract plan: \( \langle a \rangle \) (abstract cost 1)
- This is also a concrete plan (concrete cost 3)
- But optimal concrete plan: \( \langle b, a \rangle \) (concrete and abstract cost 2)
Empirical Evaluation

Experiment 1: Compare Anytime Behaviour of $h^{cegar}$ and $h^{ids}$

Setting: IPPC benchmarks, Prost planner

**ACADEMIC ADVISING**

- **Optimum**
- $h^{cegar}$
- $h^{ids}$

**TRIANGLE TIREWORLD**

- **Optimum**
- $h^{cegar}$
- $h^{ids}$

**TAMARISK**

- **Optimum**
- $h^{cegar}$
- $h^{ids}$

**SYSADMIN**

- **Optimum**
- $h^{cegar}$
- $h^{ids}$
Empirical Evaluation

Experiment 2: Compare Accuracy of $h^{cegar}$ and $h^{add}$

Observation/Question: $h^{add}$ neither admissible nor anytime, but possibly more accurate than $h^{cegar}$? Let’s see ...

Conclusion:

- $h^{cegar}$ never overestimates.
- $h^{cegar}$ becomes more accurate over time.
- After sufficient time, accuracy of $h^{cegar}$ comparable to that of $h^{add}$. 
Summary

Our motivating challenges were:

- **Understand** when abstract costs are **efficiently computable**.
  - ✔️ largely understood: if (and only if) abstraction is Cartesian

- **Make** abstraction heuristics **state-dependent-action-cost aware**.
  - ✔️ done: defined/generalized
    - Cartesian abstractions
    - local EVMDD evaluation
    - generalized CEGAR

- **Perform optimal planning** with state-dependent action costs.
  - ✔️ done: promising empirical results