On the Relationship Between State-Dependent Action Costs and Conditional Effects in Planning

Robert Mattmüller  Florian Geißer
Benedict Wright  Bernhard Nebel

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Relaxations with State-Dependent Costs and Effects

\[ x = \begin{cases} 0, & 1, & 2, & 3, & 4, & 5 \end{cases} \]

\[ \text{cost}(x) = \begin{cases} 1, & 2, & 3, & 4, & 5 \end{cases} \]

\[ \text{eff}(x) = \begin{cases} 0, & 1, & 2, & 3 \end{cases} \]

\[ h^*(x) \]

\[ x = 0 + 1 + 2 + 3 + 4 + 5 = 15 \]
Relaxations with State-Dependent Costs and Effects

\[ x = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

\[ \text{cost}(\rightarrow) = x + 1 \]
\[ \text{eff}(\rightarrow) = x' := x + 1 \]
Relaxations with State-Dependent Costs and Effects

\[
x = \begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\[
\begin{array}{c}
\text{cost}(\rightarrow) = x + 1 \\
\text{eff}(\rightarrow) = x' := x + 1 \\
h^*(x = 0) = 1 + 2 + 3 + 4 + 5 = 15
\end{array}
\]
Relaxations with State-Dependent Costs and Effects

\[
x = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}
\]

\[
\begin{array}{c}
cost(\rightarrow) = x + 1 \\
eff(\rightarrow) = x' := x + 1 \\
h^*(x = 0) = 1 + 2 + 3 + 4 + 5 = 15
\end{array}
\]
Relaxations with State-Dependent Costs and Effects

\[x = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5\]

\[x^+ = \{0, 1, 2\}\]

\[
cost(\rightarrow) = x + 1
\]
\[
eff(\rightarrow) = x' := x + 1
\]
\[
h^*(x = 0) = 1 + 2 + 3 + 4 + 5 = 15
\]
Relaxations with State-Dependent Costs and Effects

\[ x = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

\[ x^+ = \{0, 1, 2\} \]

\[ \text{cost}(\rightarrow) : \min\{1, 2, 3\} \]

\[ \text{eff}(\rightarrow) : \quad x^+ := \{0\} \cup \{1\} \cup \{2\} \cup \{3\} \]

\[ \text{cost}(\rightarrow) = x + 1 \]

\[ \text{eff}(\rightarrow) = x' := x + 1 \]

\[ h^*(x = 0) = 1 + 2 + 3 + 4 + 5 = 15 \]
Relaxations with State-Dependent Costs and Effects

\[
x^+ = \{0, 1, 2\}
\]

\[
cost(\rightarrow) : 1
\]

\[
eff(\rightarrow) : x^+' := \{0, 1, 2, 3\}
\]

\[
cost(\rightarrow) = x + 1
\]

\[
eff(\rightarrow) = x' := x + 1
\]

\[
h^*(x = 0) = 1 + 2 + 3 + 4 + 5 = 15
\]
Relaxations with State-Dependent Costs and Effects

- cost-effect mismatch!
- \( \sim \rightarrow \) uninformative heuristic
  - \( h^+(x = 0) = 5 \) vs.
  - \( h^*(x = 0) = 15 \)
Relaxations with State-Dependent Costs and Effects

**Question:** what went wrong?
Relaxations with State-Dependent Costs and Effects

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**Answer:** handled costs and effects separately.
Relaxations with State-Dependent Costs and Effects

**Question:** what went wrong?

**Answer:** handled costs and effects separately.

**Proposal:** handle uniformly!

- “compact” representation: DD exploiting additive separability
- edge-valued multi-valued decision diagrams (EVMDDs)
  (Ciardo and Siminiceanu 2002; Lai, Pedram, Vrudhula 1996)
Uniform Representation using EVMDDs

EVMDD for StateDependentCosts \((x + 1)\)
Uniform Representation using EVMDDs

EVMDD for \(ConditionalEffects\) (\(x' := x + 1\))
Uniform Representation using EVMDDs

Combined EVMDD

Consequence: effect $x'$ associated with cost $3$.
Uniform Representation using EVMDDs

Combined EVMDD

Consequence: effect $x' := 3$ now associated with cost 3.
EVMDD Construction

Next: how to construct those EVMDDs?

EVMDD Construction for Costs

Example (Multivariate Polynomial)

\[ x + y + z + yz + 1 \]
EVMDD Construction for Costs

Example (Multivariate Polynomial)

\[ x + y + z + yz + 1 \]

\[ x + y + z + yz \]
EVMDD Construction for Costs

Example (Multivariate Polynomial)

\[ x + y + z + yz + 1 \]

\[ x + y + z + yz \]

\[ y + z + yz \]
EVMDD Construction for Costs

Example (Multivariate Polynomial)

\[ x + y + z + yz + 1 \]
EVMDD Construction for Costs

Example (Multivariate Polynomial)
EVMDD Construction for Costs

Example (Multivariate Polynomial)

$x + y + z + yz + 1$

$x + y + z + yz$

$y + z + yz$

$z$

$2z$
EVMDD Construction for Costs

Example (Multivariate Polynomial)

\[ x + y + z + yz + 1 \]
EVMDD Construction for Costs

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EVMDD Construction for Costs

Example (Multivariate Polynomial)

\[ x + y + z + yz + 1 \]

\[ x + y + z + yz \]

\[ y + z + yz \]

\[ z \]

\[ 2z \]
Correct Representation of Costs

Proposition

Such cost EVMDDs correctly encode cost functions.
EVMDD Construction for Effects

Example (Effect in Normal Form, Rintanen 2003)

\[
(\neg x \triangleright \neg v') \land (x \triangleright u') \land \\
((x \lor y) \triangleright \neg z') \land ((x \land z) \triangleright v') \land w'
\]
EVMDD Construction for Effects

Example (Effect in Normal Form, Rintanen 2003)

\[
\begin{align*}
(\neg x \triangleright \neg v') & \land (x \triangleright u') \land \\
((x \lor y) \triangleright \neg z') & \land ((x \land z) \triangleright v') \land w'
\end{align*}
\]
EVMDD Construction for Effects

Example (Effect in Normal Form, Rintanen 2003)

\[
(-x \triangleright -v') \land (x \triangleright u') \land \\
((x \lor y) \triangleright -z') \land ((x \land z) \triangleright v') \land w' \\
\]

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(-x \triangleright -v') \land (x \triangleright u') \land \\
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EVMDD Construction for Effects

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EVMDD Construction for Effects

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EVMDD Construction for Effects

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EVMDD Construction for Effects

Example (Effect in Normal Form, Rintanen 2003)
EVMDD Construction for Effects

Example (Effect in Normal Form, Rintanen 2003)

\[
\begin{align*}
(\neg x) \land (x) \land (y) \land (z) \land (u) \land (v)
\end{align*}
\]
Correct Representation of Effects

**Proposition**

Such effect EVMDDs correctly encode semantics of conditional effects (= sets of active effects (Rintanen 2003)).
EVMDD Product Construction

Product EVMDD construction:

- **Option 1**: top-down construction in product space (straightforward)
- **Option 2**: product of cost and effect EVMDD
EVMDD Product Construction

Example (option 2, cost and effect EVMDDs)
EVMDD Product Construction

Example (option 2, after step 1)
EVMDD Product Construction

Example (option 2, cost-effect product)
Properties of the Construction

Claim: Product construction does the right thing.

Proposition

If
- EVMDD $\mathcal{E}_1$ represents function $f_1$ and
- EVMDD $\mathcal{E}_2$ represents function $f_2$,

then
- EVMDD $\mathcal{E}_1 \otimes \mathcal{E}_2$ represents function $f(s) = (f_1(s), f_2(s))$. □
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Claim: Product construction does the right thing.

Proposition

If

- EVMDD $E_1$ represents function $f_1$ and
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then

- EVMDD $E_1 \otimes E_2$ represents function $f(s) = (f_1(s), f_2(s))$.

Advantage: need only generic apply procedure
Properties of the Construction

Size of product EVMDD:

- **worst case**: product of factor sizes
- **best cases**: \( \max/\sum \) of factor sizes if
  - factors have identical structure:
  
  ![Diagram of identical factors](image)

  \( \otimes \)  \( \otimes \)  \( = \)  

  ![Diagram of non-identical factors](image)

  \( \otimes \)  \( = \)
Relaxed Operator Semantics under State-Dependent Costs and Effects

Definition (relaxed active effects with associated costs)

Given:
- relaxed state $s^+$,
- effect $eff$, and
- cost function $c : S \rightarrow \mathbb{N}$.

Then: the change set $[eff]_c^{s^+}$ is the set of facts that $eff$ makes true in $s^+$ together with the cheapest possible cost of doing so in $s^+$. 
Relaxed Operator Semantics under State-Dependent Costs and Effects

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Example:

$$[x' := x + 1]_{\{(x,0),(x,1),(x,2)\}}^{x+1} = \{(x' := 1, 1),
\quad (x' := 2, 2),
\quad (x' := 3, 3)\}$$
Relaxed Operator Semantics under State-Dependent Costs and Effects

**Problem:** efficient computation of $[eff]^c_{s^+}$. 

Solution: topsort traversal of cost-effect product EVMDD. 

*fact made true on some edge $\Rightarrow$ store cost 

*fact made true along different paths $\Rightarrow$ only keep cheapest cost 

*note: not just independent $\sum/\max$! 

*linear-time in EVMDD size
Relaxed Operator Semantics under State-Dependent Costs and Effects

Problem: efficient computation of $[\text{eff}]^c_{s^+}$.

Solution: topsort traversal of cost-effect product EVMDD.

- fact made true on some edge $\leadsto$ store cost
- fact made true along different paths $\leadsto$ only keep cheapest cost
- note: not just independent $\sum / \max$!
- linear-time in EVMDD size
Properties of the Computation

Proposition

The EVMDD-based change set computation computes $[\text{eff}]_{s^+}^c$. 

Summary

- informative heuristics necessitate ...
- ...uniform treatment of state-dependent costs and effects.
- representation: cost and effect EVMDDs
- construction: cofactor expansions, product EVMDD via apply
- \(\leadsto\) relaxed semantics computable using product EVMDDs
Discussion and Future Work

Discussion:

- effect EVMDD related to Fast Downward successor generator
- also: cf. Nebel’s compilation of conditional effects (2000)

Future Work:

- define and compute relaxation and abstraction heuristics with state-dependent costs and effects
- define compilation based on product EVMDDs
- variable orderings and EVMDD relaxations